

A Five-brane Modulus in the Effective N=1 Supergravity of M-Theory*

J.-P. Derendinger and R. Sauser[†]

*Institute of Physics, University of Neuchâtel
CH-2000 Neuchâtel, Switzerland*

Abstract

Five-branes lead in four dimensions to massless $N = 1$ supermultiplets if M-theory is compactified on $S^1/\mathbf{Z}_2 \times$ (a Calabi-Yau threefold). One of them describes the modulus associated with the position of the five-brane along the circle S^1 . We derive the effective four-dimensional supergravity of this multiplet and its coupling to bulk moduli and to Yang-Mills and charged matter multiplets located on \mathbf{Z}_2 fixed planes. The dynamics of the five-brane modes is obtained by reduction and supersymmetrization of the covariant five-brane bosonic action. Our construction respects all symmetries of M-theory, including the self-duality of the brane antisymmetric tensor. Corrections to gauge couplings are strongly constrained by this self-duality property. The brane contribution to the effective scalar potential is formally similar to a renormalization of the dilaton. The vacuum structure is not modified. Altogether, the impact of the five-brane modulus on the effective supergravity is reminiscent of string one-loop corrections produced by standard compactification moduli.

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[†] jean-pierre.derendinger, roger.sauser@unine.ch

1 Introduction

Compactification of M-theory on

$$O_7 = X_6 \times S^1/\mathbf{Z}_2, \quad (1.1)$$

with a Calabi-Yau threefold X_6 , leads to $N = 1$ supersymmetry in four space-time dimensions. Five-branes configurations preserve this supersymmetry if their world-volume \mathcal{W}_6 is suitably aligned: it should enclose four-dimensional Minkowski space \mathcal{M}_4 and a holomorphic two-cycle \mathcal{C}_2 in X_6 [1, 2, 3]: $\mathcal{W}_6 = \mathcal{M}_4 \times \mathcal{C}_2$. With this embedding, five-brane massless excitations on the world-volume, which belong to a tensor multiplet of chiral six-dimensional supersymmetry on \mathcal{W}_6 [4, 5], produce in the low-energy four-dimensional effective supergravity various $N = 1$ multiplets of massless fields. Some of these modes are deeply related to the Calabi-Yau geometry, and computing their effective theory is a very complicated task. There are however universal modes which can be more easily described, the most obvious example being the real scalar associated to the position of the five-brane on the orbifold S^1/\mathbf{Z}_2 . This ‘universal five-brane modulus’ will be the main subject of the present paper: we will compute its effective supergravity couplings to the modes of M-theory on O_7 which are also perturbative massless states of $E_8 \times E_8$ heterotic strings on X_6 . In the simplest case of the standard embedding, these are the $N = 1$ supergravity and dilaton multiplets, the modulus of the Calabi-Yau volume, $E_6 \times E_8$ gauge fields and chiral matter in representation $(\mathbf{27}, \mathbf{1})$. Lukas, Ovrut and Waldram (LOW) [6] have derived the effective supergravity for these heterotic states in a non-trivial background value of the brane modulus. Our goal here is to obtain a complete¹ effective supergravity for the supermultiplet of the universal brane modulus.

When computing an effective Lagrangian, it is usually important to respect the symmetries of the underlying theory. For instance, the tensor multiplet of five-brane excitations has an antisymmetric tensor with a self-dual field strength. This symmetry has important implications in four dimensions: the effective theory has a massless antisymmetric tensor dual to a pseudoscalar or, in terms of supermultiplets a chiral multiplet dual to a linear multiplet² [7]. This observation has immediate implications on the effective supergravity of the brane modes since only a limited class of chiral

¹Including terms with two derivatives or less.

²This property is called ‘chiral-linear duality’ in the paper.

multiplets couplings is allowed by chiral-linear duality [8]. Another example is the fact that M-theory on O_7 can be defined by specific Bianchi identities. Their symmetry properties provide information on the supergravity multiplets to be used in their effective description. In ref. [9], we have formulated the effective supergravity of M-theory on O_7 without five-branes using Lagrange multiplier superfields to impose, by their field equations, these Bianchi identities and all their symmetries. This formulation is well adapted to the inclusion of five-brane modes.

The construction reveals some interesting features. The contributions of the five-brane universal modulus are closely similar to the perturbative corrections generated by volume moduli [10, 11, 12]. In particular, gauge threshold corrections arise, with a gauge-group independent term linked by supersymmetry to brane kinetic terms. This correction can be regarded as a renormalization of the dilaton field. Of course, there is no induced superpotential and the vacuum properties of the scalar potential are not severely modified. The physics impact of the five-brane fields is in the modification of the M-theory background equation (the ‘cohomology condition’ [3]) and in the gauge-group-dependent threshold corrections, as observed by LOW [6].

The present article is divided in three parts. In section 2, we study the role of the six-form field which couples naturally to the five-brane. We construct a version of the bosonic sector of eleven-dimensional supergravity in which the field equation of the six-form field is the required Bianchi identity. This theory can then easily be coupled to contributions arising from S^1/\mathbf{Z}_2 fixed planes or from five-branes. Its reduction on O_7 provides the link with the effective four-dimensional supergravity derived in ref. [9] and a guiding line for the introduction of five-brane fields. The section ends with a first glance at the Calabi-Yau background equation.

Section 3 is devoted to the dynamics of the five-brane massless modes. Our starting point is the self-dual formulation of the bosonic five-brane action, with an auxiliary scalar, as derived by Pasti, Sorokin and Tonin (PST) [13]. The O_7 truncation is performed and supersymmetrized, in flat space and in an eleven-dimensional supergravity background. The resulting kinetic Lagrangian for the five-brane modulus multiplet possesses as expected chiral-linear duality: the brane modulus can be either described by a linear supermultiplet \hat{L} or by a chiral \hat{S} with symmetry $\hat{S} \longrightarrow \hat{S} + ic$ (c a real constant). This invariance severely restricts the possible form of the brane contributions in the Lagrangian. We also discuss how the various contributions to the scalar

potential cancel each other.

The complete effective supergravity coupled to the five-brane Lagrangian is the subject of section 4. Following the procedure valid for the Calabi-Yau volume modulus T , we introduce threshold corrections as the most general term allowed by the shift symmetry acting on the brane multiplet \hat{S} . We then consider the two dual versions of the effective supergravity, with the dilaton embedded either in a chiral or in a linear multiplet. The analysis of the gauge couplings in the linear version reveals a universal quadratic correction generated by the brane kinetic terms, and a linear dependence in the threshold terms. In the chiral version of the theory, the quadratic correction is moved into the Kähler potential of the chiral dilaton $S + \overline{S}$. This result is strongly similar to standard gauge threshold corrections in the modulus T , which are perturbative one-loop contributions in string theory. We then compare our expressions with the background found by LOW and discuss the modifications of the scalar potential introduced by the brane modulus.

Finally, section 5 contains some concluding remarks and an appendix defines our conventions³.

2 The six-form field

In this section, we first discuss a formulation of the bosonic sector of eleven-dimensional supergravity in which the Bianchi identity for the four-form G_4 is explicitly given by the field equation of a six-form C_6 . This eleven-dimensional field plays the role of a multiplier and its Lagrangian can be easily modified to include source contributions arising, for instance, from five-branes⁴. Explicitly, the modified Lagrangian is of the form $C_6 \wedge (dG_4 - \Delta_5)$, where Δ_5 is the five-form source of the Bianchi identity. It also turns out to be at the origin of the four-dimensional ‘Lagrange multiplets’ described in a previous publication [9], in which Bianchi identities were field equations. After having introduced C_6 at the level of the bosonic sector of the standard Cremmer-Julia-Scherk eleven-dimensional supergravity [16], we consider the modifications required by the two ten-dimensional planes fixed under \mathbf{Z}_2 and by the presence of five-branes.

³ We use the same notations as in ref. [9] and our conventions for supergravity expressions are (mostly) as in ref. [14].

⁴ Our procedure is similar, but not identical, to the method of de Alwis [15]

2.1 Eleven-dimensional supergravity

We begin by considering the standard CJS formulation [16]. In terms of differential forms, the bosonic part of the eleven-dimensional supergravity action is given by

$$2\kappa_{11}^2 S_{\text{CJS}} = - \int_{\mathcal{M}_{11}} eR - \frac{1}{2} \int_{\mathcal{M}_{11}} G_4 \wedge *G_4 - \frac{1}{6} \int_{\mathcal{M}_{11}} C_3 \wedge G_4 \wedge G_4, \quad (2.1)$$

where the two independent fields are the metric (vielbein) and the three-form potential C_3 . The four-form field strength G_4 is defined by $G_4 = dC_3$, and \mathcal{M}_{11} is eleven-dimensional Minkowski space. The equation of motion for C_3 that can be computed from the action (2.1) is

$$C_3 : d *G_4 = -\frac{1}{2} G_4 \wedge G_4, \quad (2.2)$$

and the Bianchi identity reads $dG_4 = 0$. Note that S_{CJS} is invariant under the standard gauge transformation

$$C_3 \rightarrow C_3 + d\Lambda_2, \quad (2.3)$$

where Λ_2 is a two-form.

Since we would like to incorporate “magnetic” five-branes⁵ in our discussion, it is natural to look for an action which contains a seven-form field strength G_7 dual to the usual four-form G_4 . The structure of the topological term $C_3 \wedge G_4 \wedge G_4$ in (2.1) does not allow us to completely eliminate the three-form C_3 , and an action trivially equivalent to S_{CJS} is

$$\begin{aligned} 2\kappa_{11}^2 S_{11\text{sd}} = & - \int_{\mathcal{M}_{11}} eR - \frac{1}{2} \int_{\mathcal{M}_{11}} G_4 \wedge *G_4 - \frac{1}{6} \int_{\mathcal{M}_{11}} C_3 \wedge dC_3 \wedge dC_3 \\ & + \int_{\mathcal{M}_{11}} G_7 \wedge (G_4 - dC_3), \end{aligned} \quad (2.4)$$

where the four independent fields are now the metric (vielbein), the three-form C_3 , the four-form G_4 and the seven-form G_7 . The equations of motion for the antisymmetric tensor fields are

$$\begin{aligned} C_3 : dG_7 &= -\frac{1}{2} dC_3 \wedge dC_3, \\ G_4 : *G_4 &= G_7, \\ G_7 : G_4 &= dC_3. \end{aligned} \quad (2.5)$$

They are certainly equivalent to the original field equation in the CJS version of the theory. The solution of the equation for C_3 is

$$G_7 = -dC_3 - \frac{1}{2} C_3 \wedge dC_3, \quad (2.6)$$

⁵ As opposed to the “electric” membranes which naturally couple to the CJS action (2.1).

where C_6 is an arbitrary six-form potential. Notice that the invariance of the seven-form G_7 under the gauge transformation (2.3) imposes that

$$C_6 \rightarrow C_6 - \frac{1}{2}\Lambda_2 \wedge dC_3 + d\Lambda_5. \quad (2.7)$$

We can now write a more interesting form of the bosonic sector of eleven-dimensional supergravity in which the Bianchi identity is imposed via by six-form field C_6 . Substituting the expression (2.6) for G_7 into the action (2.4), we obtain (with a partial integration) a formulation where the four independent fields are the metric, C_3 , G_4 and C_6 :

$$\begin{aligned} 2\kappa_{11}^2 S_{11\text{sd}'} = & - \int_{\mathcal{M}_{11}} eR - \frac{1}{2} \int_{\mathcal{M}_{11}} G_4 \wedge *G_4 - \frac{1}{2} \int_{\mathcal{M}_{11}} C_3 \wedge dC_3 \wedge (G_4 - \frac{2}{3}dC_3) \\ & + \int_{\mathcal{M}_{11}} C_6 \wedge dG_4. \end{aligned} \quad (2.8)$$

This action is invariant under gauge symmetries (2.3) and (2.7). The equations of motion for G_4 , C_6 and C_3 are now

$$\begin{aligned} G_4 : \quad & *G_4 = -dC_6 - \frac{1}{2}C_3 \wedge dC_3, \\ C_6 : \quad & dG_4 = 0, \\ C_3 : \quad & dC_3 \wedge (dC_3 - G_4) = -\frac{1}{2}C_3 \wedge dG_4. \end{aligned} \quad (2.9)$$

The exterior derivative of the first equation is the CJS equation (2.2) if in addition $G_4 = dC_3$. The second relation is the Bianchi identity which says that locally $G_4 = dA_3$. Finally, the third equation implies that C_3 and A_3 can differ by a gauge transformation (2.3) and by irrelevant particular solution to eq. (2.2).

The O_7 truncation of theory (2.8) is as follows. Under \mathbf{Z}_2 , C_3 and G_4 are as usual odd while C_6 is even. Since \mathbf{Z}_2 acts on the S^1 coordinate x^4 , the universal massless modes of C_6 surviving the truncation will be $C_{\mu\nu\rho\sigma i\bar{j}}$, $C_{\mu\nu i\bar{j}i\bar{j}}$, $C_{ijk\bar{i}\bar{j}\bar{k}}$ and $C_{\mu\nu\rho ijk}$ (as well as the conjugate $C_{\mu\nu\rho i\bar{j}\bar{k}} = C_{\mu\nu\rho ijk}^*$). Their field equations respectively imply the Bianchi identity for $(dG_4)_{4i\bar{j}i\bar{j}}$ which is a background equation since it is not associated to any four-dimensional massless mode, and the four-dimensional Bianchi identities for the massless components $G_{4\rho k\bar{k}}$, $G_{4\mu\nu\rho}$ and $G_{4i\bar{j}\bar{k}}$. Notice also that the modified topological term in action (2.8) is eliminated by the truncation. The resulting truncated four-dimensional action is trivial as long as contributions from \mathbf{Z}_2 fixed planes and five-branes are not included.

2.2 Orbifold and five-brane contributions

If one assumes that the Bianchi identity is not $dG_4 = 0$, but instead $dG_4 = \Delta_5$ with an exact five-form $\Delta_5 = d\Delta_4$ not depending on C_3 , G_4 or C_6 , the action (2.8) can be consistently modified to become:

$$2\kappa_{11}^2 S = -\frac{1}{2} \int_{\mathcal{M}_{11}} G_4 \wedge *G_4 - \frac{1}{2} \int_{\mathcal{M}_{11}} C_3 \wedge dC_3 \wedge (G_4 - \Delta_4 - \frac{2}{3}dC_3) + \int_{\mathcal{M}_{11}} C_6 \wedge (dG_4 - \Delta_5) + \text{Einstein term} + \dots \quad (2.10)$$

The independent fields are C_3 , G_4 and C_6 . The term with C_6 is modified to obtain the new Bianchi identity with source Δ_5 . The additional $C_3 \wedge dC_3 \wedge \Delta_4$ term is a possible addition to cancel the variation under (2.3) of the source contribution $-C_6 \wedge \Delta_5$. We will however see below that if a five-brane is at the origin of the source Δ_5 , another modification arises. The dots in action (2.10) denote possible contributions which do not involve the eleven-dimensional bulk fields and are related to the dynamics of the magnetic source Δ_5 . An example would be the ten-dimensional kinetic terms for the gauge fields in a compactification of M-theory on S^1/\mathbf{Z}_2 . The equations of motion for C_3 , G_4 and C_6 are

$$\begin{aligned} C_3 &: dC_3 \wedge (dC_3 - G_4 + \Delta_4) = -\frac{1}{2}C_3 \wedge (dG_4 - \Delta_5), \\ G_4 &: *G_4 = -dC_6 - \frac{1}{2}C_3 \wedge dC_3, \\ C_6 &: dG_4 = \Delta_5. \end{aligned} \quad (2.11)$$

Compactification of M-theory on O_7 or S^1/\mathbf{Z}_2 has two kinds of defects generating sources Δ_5 : M-theory five-branes and \mathbf{Z}_2 fixed planes. A tensor $N = 2$ multiplet of massless excitations lives on the world-volume of a five-brane [4, 5] and a E_8 super-Yang-Mills multiplet is located on each fixed plane [3, 17, 18].

A five-brane in an eleven-dimensional background can be described by the following world-volume bosonic action [19]

$$S_{M5} = \int_{\mathcal{W}_6} \mathcal{L}_{\text{kin.}} - T_5 \int_{\mathcal{W}_6} \hat{C}_6 - \frac{T_5}{2} \int_{\mathcal{W}_6} \hat{C}_3 \wedge d\mathcal{B}_2, \quad (2.12)$$

where the hatted fields \hat{C}_3 and \hat{C}_6 are the eleven-dimensional background fields pulled back onto the six-dimensional world-volume \mathcal{W}_6 and the two-form \mathcal{B}_2 belongs to the $D = 6$ supermultiplet of the five-brane⁶. The first contribution $\mathcal{L}_{\text{kin.}}$ describes the

⁶ See Section 3 for more detail on this multiplet.

kinematics of the bosonic degrees of freedom. It includes a Born-Infeld term for the induced metric tensor $\hat{g}_{\hat{m}\hat{n}}$ coupled to the three-form $H_3 \equiv d\mathcal{B}_2 - \hat{C}_3$, which is submitted to a self-duality condition. In the covariant formalism of Pasti, Sorokin and Tonin [20], this self-duality condition is generated by an auxiliary scalar field. Hence, $\mathcal{L}_{\text{kin.}}$ is a functional of $\hat{g}_{\hat{m}\hat{n}}$, H_3 and of the auxiliary PST scalar, but its precise form is unimportant for a while. Notice that invariance under (2.3) of H_3 implies $\delta\mathcal{B}_2 = \hat{\Lambda}_2$, and that, with this transformation of \mathcal{B}_2 , the complete action S_{M5} is gauge invariant.

The five-brane action S_{M5} includes the C_6 term

$$-T_5 \int_{\mathcal{W}_6} \hat{C}_6 = -T_5 \int_{\mathcal{M}_{11}} C_6 \wedge \delta_5, \quad (2.13)$$

where the equality for an arbitrary six-form would define the closed delta function five-form δ_5 . Comparison with the C_6 term in action (2.10) indicates that adding a five-brane contribution modifies the source Δ_5 according to $\Delta_5 \rightarrow \Delta_5 + 2\kappa_{11}^2 T_5 \delta_5$, without however affecting Δ_4 since gauge invariance is obtained with the new contribution $-\frac{T_5}{2} \int_{\mathcal{M}_{11}} C_3 \wedge d\mathcal{B}_2 \wedge \delta_5$.

Using δ_5 to rewrite the action (2.12) as

$$S_{\text{M5}} = \int_{\mathcal{M}_{11}} \mathcal{L}_{\text{kin.}} \wedge \delta_5 - T_5 \int_{\mathcal{M}_{11}} C_6 \wedge \delta_5 - \frac{T_5}{2} \int_{\mathcal{M}_{11}} C_3 \wedge d\mathcal{B}_2 \wedge \delta_5, \quad (2.14)$$

we obtain a complete action from which the modified Bianchi identity with a five-brane source added can be deduced as an equation of motion for the six-form C_6 :

$$\begin{aligned} 2\kappa_{11}^2 S &= -\frac{1}{2} \int_{\mathcal{M}_{11}} G_4 \wedge *G_4 - \frac{1}{2} \int_{\mathcal{M}_{11}} C_3 \wedge dC_3 \wedge (G_4 - \Delta_4 - \frac{2}{3}dC_3) \\ &\quad + \int_{\mathcal{M}_{11}} C_6 \wedge (dG_4 - \Delta_5 - 2\kappa_{11}^2 T_5 \delta_5) \\ &\quad + 2\kappa_{11}^2 \int_{\mathcal{M}_{11}} \mathcal{L}_{\text{kin.}} \wedge \delta_5 - \kappa_{11}^2 T_5 \int_{\mathcal{M}_{11}} C_3 \wedge d\mathcal{B}_2 \wedge \delta_5 + \text{Einstein term} + \dots, \end{aligned} \quad (2.15)$$

where the independent fields are the metric, C_3 , G_4 , C_6 and \mathcal{B}_2 ⁷. Their equations of motion are

$$\begin{aligned} C_3 &: dC_3 \wedge (dC_3 - G_4 + \Delta_4) = -2\kappa_{11}^2 \left(\frac{\delta \mathcal{L}_{\text{kin.}}}{\delta C_3} - \frac{T_5}{2} d\mathcal{B}_2 \right) \wedge \delta_5 - \frac{1}{2} C_3 \wedge (dG_4 - \Delta_5), \\ G_4 &: *G_4 = -dC_6 - \frac{1}{2} C_3 \wedge dC_3, \\ C_6 &: dG_4 = \Delta_5 + 2\kappa_{11}^2 T_5 \delta_5, \\ \mathcal{B}_2 &: \frac{\delta \mathcal{L}_{\text{kin.}}}{\delta \mathcal{B}_2} \wedge \delta_5 = \frac{T_5}{2} dC_3 \wedge \delta_5. \end{aligned} \quad (2.16)$$

⁷As well as the translational degrees of freedom of the five-brane world-volume, in the pull-back of \mathcal{M}_{11} onto \mathcal{W}_6 .

Taking the exterior derivative of the first equation and using the equality $d(\frac{\delta \mathcal{L}_{\text{kin.}}}{\delta C_3}) = \frac{\delta \mathcal{L}_{\text{kin.}}}{\delta \mathcal{B}_2}$ which follows from the fact that C_3 and \mathcal{B}_2 only appear through H_3 in $\mathcal{L}_{\text{kin.}}$, we recover the last equation when the third relation is taken into account.

This discussion can be easily extended to the presence of several five-branes. In the case of M-theory on O_7 , five-brane world-volumes must be embedded in \mathcal{M}_{11} in a \mathbf{Z}_2 -invariant way.

We now proceed to add the contributions due to \mathbf{Z}_2 fixed planes. They will correspond to specific expressions for the source Δ_5 and its primitive Δ_4 in action (2.15). And since these expressions do not depend on the eleven-dimensional or five-brane fields, the equations (2.16) and their significance will remain unchanged.

In the presence of five-branes, M-theory on $S^1/\mathbf{Z}_2 \times \mathcal{M}_{10}$ can be defined by the following Bianchi identity [3, 17, 18, 21]:

$$dG_4 = -(4\pi)^2 \frac{\kappa_{11}^2}{\lambda^2} \left[I_{4,1} \wedge \delta_{1,1} + I_{4,2} \wedge \delta_{1,2} + \sum_j \delta_5(\mathcal{W}_{6,j}) \right], \quad (2.17)$$

where $\mathcal{W}_{6,j}$ is the world-volume of the j 'th five-brane and $\delta_5(\mathcal{W}_{6,j})$ the corresponding five-form as defined in eq. (2.13). The S^1/\mathbf{Z}_2 direction x^4 has periodicity 2π , \mathbf{Z}_2 acts according to $x^4 \rightarrow -x^4$ and fixed points are at $x^4 = 0$ and π . For each five brane with world-volume $\mathcal{W}_{6,j}$, there exists a five-brane with world-volume given by the image under \mathbf{Z}_2 of $\mathcal{W}_{6,j}$. Eq. (2.17) also gives the expression $T_5 = -8\pi^2/\lambda^2$ in terms of the gauge coupling constant λ on the ten-dimensional fixed planes. The Dirac one-forms on S^1 read

$$\delta_{1,1} = \delta(x^4) dx^4, \quad \delta_{1,2} = \delta(x^4 - \pi) dx^4. \quad (2.18)$$

Finally, on the ten-dimensional \mathbf{Z}_2 fixed planes, at $x^4 = 0$ and $x^4 = \pi$, live four-forms

$$I_{4,i} = \frac{1}{(4\pi)^2} \left[\text{tr } F_i^2 - \frac{1}{2} \text{tr } R^2 \right], \quad i = 1, 2, \quad (2.19)$$

where each F_i is an E_8 gauge curvature and R is Riemann curvature. We then conclude that the appropriate bosonic action for M-theory on S^1/\mathbf{Z}_2 can be written as:

$$\begin{aligned} S &= \int_{\mathcal{M}_{11}} \mathcal{L}, \\ 2\kappa_{11}^2 \mathcal{L} &= -\frac{1}{2} G_4 \wedge *G_4 - \frac{1}{2} C_3 \wedge dC_3 \wedge (G_4 - \Delta_4 - \frac{2}{3} dC_3) \\ &\quad + C_6 \wedge \left(dG_4 + (4\pi)^2 \frac{\kappa_{11}^2}{\lambda^2} \left[I_{4,1} \wedge \delta_{1,1} + I_{4,2} \wedge \delta_{1,2} + \sum_j \delta_5(\mathcal{W}_{6,j}) \right] \right) \\ &\quad + 2\kappa_{11}^2 \sum_j \left(\mathcal{L}_{\text{kin.}}(H_{3(j)}) + \frac{4\pi^2}{\lambda^2} C_3 \wedge d\mathcal{B}_{2(j)} \right) \wedge \delta_5(\mathcal{W}_{6,j}) \\ &\quad - \frac{\kappa_{11}^2}{\lambda^2} \left(F_1 \wedge *F_1 \wedge \delta_{1,1} + F_2 \wedge *F_2 \wedge \delta_{1,2} \right) + \text{Einstein term}. \end{aligned} \quad (2.20)$$

The last line includes the kinetic terms of the E_8 gauge fields living on each fixed plane and the four-form Δ_4 is defined as the solution to the Bianchi identity (2.17) *without any five-brane*:

$$d\Delta_4 = -(4\pi)^2 \frac{\kappa_{11}^2}{\lambda^2} \left[I_{4,1} \wedge \delta_{1,1} + I_{4,2} \wedge \delta_{1,2} \right]. \quad (2.21)$$

Notice that each five-brane has its own tensor $H_{3(j)} = d\mathcal{B}_{2(j)} - C_3$, up to the identification of a five-brane with its image under \mathbf{Z}_2 .

It should be remarked that the theory (2.20) is not equivalent to the Hořava-Witten action. It is a generalization of the bosonic sector of eleven-dimensional supergravity and it does not include an anomaly-cancelling term similar to the contribution $\int_{\mathcal{M}_{11}} C_3 \wedge G_4 \wedge G_4$. Cancellation of chiral anomalies requires the addition of appropriate Green-Schwarz counterterms.

2.3 The background

The Bianchi identities of M-theory compactified on O_7 are the components of eq. (2.17) reduced on the Calabi-Yau space. They are also the field equations of the components of C_6 reduced on O_7 . Denoting by V_6 the Calabi-Yau volume and using $\kappa_{11}^2 = 2\pi V_6 \kappa^2$, where κ is the four-dimensional gravitational constant, one infers that the dimensionless number λ^2/V_6 can be absorbed in the metric moduli, so that these identities as well as the four-dimensional reduced action depend on a single parameter, the four-dimensional gravitational constant⁸. As mentioned earlier, the field equation of the component $C_{\mu\nu\rho\sigma i\bar{j}}$ is the background equation

$$(dG_4)_{4ij\bar{k}\bar{l}} = -(4\pi)^2 \frac{\kappa_{11}^2}{\lambda^2} \left[I_{4,1} \wedge \delta_{1,1} + I_{4,2} \wedge \delta_{1,2} + \sum_j \delta_5(\mathcal{W}_{6,j}) \right]_{4ij\bar{k}\bar{l}}. \quad (2.22)$$

This equation integrated over a closed five-cycle gives, for a globally well-defined G_4 , the standard ‘cohomology condition’ which defines the embedding of the four-dimensional gauge group into $E_8 \times E_8$ [3]. In general, it implies non-zero background values for $(\text{tr } F_1^2)_{ij\bar{k}\bar{l}}$ and/or $(\text{tr } F_2^2)_{ij\bar{k}\bar{l}}$, and relates these vacuum values to the Calabi-Yau background $(\text{tr } R^2)_{ij\bar{k}\bar{l}}$. Since there are no massless fluctuations associated with this component of dG_4 , we may assume that the fluctuation $C_{\mu\nu\rho\sigma i\bar{j}}$ is zero when computing the reduced effective Lagrangian, provided we develop the theory around the appropriate background.

⁸See ref. [9] for a detailed discussion. For the same reason, we may choose the S^1 radius to be one.

We denote the form degree on $\mathcal{M}_4 \times O_7$ as (m, n, p, q) . The degree on \mathcal{M}_4 is m , the holomorphic and anti-holomorphic degrees on the Calabi-Yau space are n and p , and q is the degree on S^1/\mathbf{Z}_2 . The $SU(3)$ holonomy condition implies that the background $\langle G_4 \rangle$ is a $(0, 2, 2, 0)$ form. The defining equation for the six-form field C_6 is the duality equation $*G_4 = -dC_6 - \frac{1}{2}C_3 \wedge dC_3$. The background $\langle dC_6 \rangle$ is then a $(4, 1, 1, 1)$ form. By $SU(3)$ holonomy and \mathbf{Z}_2 symmetry, the background component $\langle C_6 \rangle$ of C_6 is a $(4, 1, 1, 0)$ form and $\langle dC_6 \rangle = \frac{\partial}{\partial x^4} \langle C_6 \rangle dx^4$. The equations defining the background are then⁹ $\langle dC_6 \rangle = - * \langle G_4 \rangle$ and

$$\langle d * dC_6 \rangle = -(4\pi)^2 \frac{\kappa_{11}^2}{\lambda^2} \left\langle I_{4,1} \wedge \delta_{1,1} + I_{4,2} \wedge \delta_{1,2} + \sum_j \delta_5(\mathcal{W}_{6,j}) \right\rangle.$$

They depend in general of the metric tensor reduced on $\mathcal{M}_4 \times O_7$ since they use the Hodge dual and Dirac tensorial distributions. This condition has been studied in detail in refs. [22, 6].

In our approach based on Lagrangian (2.20), however, the background contribution is more involved. Since the six-form field multiplies the Bianchi identity, all C_6 background contributions automatically cancel. But the background values of $G_4 \wedge *G_4$, of the Einstein term and of the gauge and brane (Born-Infeld) kinetic terms are non-zero. We will return to this point when computing the effective four-dimensional scalar potential, which vanishes, in the next section.

Our task now is to obtain the four-dimensional Calabi-Yau reduction of the action (2.20), and to extend it to a Poincaré $N = 1$ supergravity. Without five-branes, the result is well-known either from heterotic strings on X_6 [23] or from M-theory on O_7 [24, 22], and reference [9] gives a discussion based on Bianchi identities which is also the approach followed here.

3 The M-theory five-brane

The action (2.20) includes kinetic terms for the five-brane bosonic degrees of freedom, which in particular propagate the self-dual three-form H_3 . We find useful to incorporate in our discussion the largest possible symmetry. As a consequence, we will use for these kinetic terms the formalism of Pasti, Sorokin and Tonin [20] adapted to the five-brane [13, 25], in which self-duality follows from field equations.

⁹ When reduced on $\mathcal{M}_4 \times O_7$, $\langle C_3 \wedge dC_3 \rangle$ vanishes.

Since the five-brane has also scalar fields related to the translational modes of its world-volume, we begin with a brief discussion of the embedding of a world-volume \mathcal{W}_6 in $\mathcal{M}_4 \times O_7$.

3.1 Reduction of the M-five-brane bosonic action to four-dimensions

In order to preserve $N = 1$ four-dimensional supersymmetry, each five-brane must be aligned with a world-volume $\mathcal{W}_{6,j} = \mathcal{M}_4 \times \mathcal{C}_{2,j}$, the two-cycle $\mathcal{C}_{2,j}$ in the Calabi-Yau manifold being holomorphic [1, 2, 3]. We can then choose the five-brane world-volume coordinates as

$$y^{\hat{m}} = (y^\mu, y, \bar{y}), \quad \hat{m} = \hat{0}, \hat{1}, \dots, \hat{5}, \quad \mu = 0, 1, 2, 3, \quad (3.1)$$

with a complex coordinate y along the Calabi-Yau two-cycle. The embedding of the world-volume in \mathcal{M}_{11} is defined by the functions¹⁰ $x^M(y^{\hat{m}})$, $M = 0, 1, \dots, 10$, and by the pull-back functions $\frac{\partial x^M}{\partial y^{\hat{m}}}$. In $\mathcal{M}_4 \times O_7$, we use coordinates $x^M = (x^\mu, x^4, z^i, \bar{z}^{\bar{i}})$, $i = 1, 2, 3$, with

$$x^\mu = y^\mu, \quad z^i = z^i(y^\mu, y), \quad \bar{z}^{\bar{i}} = \bar{z}^{\bar{i}}(y^\mu, \bar{y}),$$

choosing a parametrization of \mathcal{M}_4 .

The five-brane excitations are described by a $D = 6$ tensor supermultiplet [4, 5]. The fields are a chiral antisymmetric tensor $\mathcal{B}_{\hat{m}\hat{n}}$ (with a self-dual field strength $H_{\hat{m}\hat{n}\hat{p}}$), five scalar fields $X_{(1)}, \dots, X_{(5)}$ specifying the position of the world-volume \mathcal{W}_6 in \mathcal{M}_{11} , and their fermionic partners. In our $\mathcal{M}_4 \times O_7$ reduction, we neglect the detailed structure of the Calabi-Yau manifold. Of the five scalar fields, only one survives as the massless mode of the Calabi-Yau expansion of $x^4(y^\mu, y, \bar{y})$,

$$\begin{aligned} x^4(y^\mu, y, \bar{y}) &= X(x^\mu) + \text{massive modes}, \\ z^i &= z^i(y), \quad \bar{z}^{\bar{i}} = \bar{z}^{\bar{i}}(\bar{y}), \quad x^\mu = y^\mu. \end{aligned} \quad (3.2)$$

This means that we will only retain the following bosonic five-brane excitations:

$$\mathcal{B}_{\mu\nu}(x^\mu) = \mathcal{B}_{\mu\nu}(y^\mu), \quad \mathcal{B}_{4\hat{5}}(x^\mu) = \mathcal{B}_{4\hat{5}}(y^\mu), \quad X(x^\mu), \quad (3.3)$$

and the self-duality condition on $H_{\hat{m}\hat{n}\hat{p}}$ relates $\mathcal{B}_{\mu\nu}$ and $\mathcal{B}_{4\hat{5}}$. The background value of the scalar field X is the five-brane position along the S^1/\mathbf{Z}_2 orbifold direction x^4 .

¹⁰Notice that x^4 is the S^1/\mathbf{Z}_2 direction in \mathcal{M}_{11} while $y^{\hat{4}}$ (and $y^{\hat{5}}$) are in the Calabi-Yau manifold.

Each five-brane generates then in \mathcal{M}_4 two bosonic degrees of freedom. By $N = 1$ supersymmetry, they will be described by either a linear or a chiral multiplet.

With these choices of embedding and truncation, the world-volume induced metric¹¹

$$\hat{g}_{\hat{m}\hat{n}} = \frac{\partial x^M}{\partial y^{\hat{m}}} \frac{\partial x^N}{\partial y^{\hat{n}}} g_{MN}$$

reduces in four dimensions to

$$\begin{aligned}\hat{g}_{\mu\nu} &= e^{-\gamma-2\sigma} g_{\mu\nu} + e^{2\gamma-2\sigma} (\partial_\mu X)(\partial_\nu X), \\ \hat{g}_{4\hat{5}} &= k^2 e^\sigma, \\ \hat{g}_{\mu\hat{4}} &= \hat{g}_{\mu\hat{5}} = \hat{g}_{4\hat{4}} = \hat{g}_{\hat{5}\hat{5}} = 0,\end{aligned}\tag{3.4}$$

where $k^2 = \delta_{i\bar{i}} \frac{\partial z^i}{\partial y} \frac{\partial \bar{z}^{\bar{i}}}{\partial \bar{y}}$ is a constant (a background value) in our Kaluza-Klein truncation.

To describe the dynamics of the bosonic fields (3.3) and their couplings to four-dimensional supergravity, we need an action for the five-brane coupled to eleven-dimensional supergravity. Using the PST formalism to write covariant Lagrangians for self-dual (or anti-self-dual) tensors, a kappa-symmetric covariant world-volume Lagrangian for the five-brane excitations has been constructed [13, 25], completing earlier work [26, 21, 19]. In a non-trivial eleven-dimensional supergravity background, the action has two parts: a kinetic Lagrangian with a Born-Infeld term involving the three-index tensor $H_{\hat{m}\hat{n}\hat{p}}$ and a Wess-Zumino term involving both C_3 and its dual C_6 . The bosonic action is¹²:

$$\begin{aligned}S_{M5} &= T_5 \int_{\mathcal{W}_6} d^6 y \left(-\sqrt{-\det(\hat{g}_{\hat{m}\hat{n}} + i H^*_{\hat{m}\hat{n}})} - \frac{1}{4} \sqrt{-\hat{g}} \mathcal{V}_i H^{*\hat{l}\hat{m}\hat{n}} H_{\hat{m}\hat{n}\hat{p}} \mathcal{V}^{\hat{p}} \right) \\ &\quad - T_5 \int_{\mathcal{W}_6} \left(\hat{C}_6 - \frac{1}{2} d\mathcal{B}_2 \wedge \hat{C}_3 \right).\end{aligned}\tag{3.5}$$

The second line is as in eq. (2.12) and the first two terms define the kinetic Lagrangian in the PST formalism. In this expression, \hat{C}_3 and \hat{C}_6 are the eleven-dimensional background fields pulled back onto the world-volume using derivatives $\partial x^M / \partial y^{\hat{m}}$,

$$\hat{C}_{\hat{m}\hat{n}\hat{p}} = \frac{\partial x^M}{\partial y^{\hat{m}}} \frac{\partial x^N}{\partial y^{\hat{n}}} \frac{\partial x^P}{\partial y^{\hat{p}}} C_{MNP}, \quad \hat{C}_{\hat{m}_1 \dots \hat{m}_6} = \frac{\partial x^{M_1}}{\partial y^{\hat{m}_1}} \dots \frac{\partial x^{M_6}}{\partial y^{\hat{m}_6}} C_{M_1 \dots M_6}, \tag{3.6}$$

¹¹ The two-index tensor g_{MN} is the eleven-dimensional metric which was defined and used in ref. [9]. Its reduction can also be found in the appendix [eq. (A.1)].

¹² Our conventions are mostly as in ref. [25]. We consider here a single five-brane.

T_5 is the brane tension and

$$\begin{aligned}
H_{\hat{m}\hat{n}\hat{p}} &= 3\partial_{[\hat{m}}\mathcal{B}_{\hat{n}\hat{p}]} - \hat{C}_{\hat{m}\hat{n}\hat{p}}, \\
H^*_{\hat{m}\hat{n}} &= H^*_{\hat{m}\hat{n}\hat{p}}\mathcal{V}^{\hat{p}}, \\
H^*_{\hat{m}\hat{n}\hat{p}} &= -\frac{1}{3!\sqrt{-\hat{g}}}\epsilon^{\hat{m}\hat{n}\hat{p}\hat{q}\hat{r}\hat{s}}H_{\hat{q}\hat{r}\hat{s}}, \\
d\mathcal{B}_2 &= \frac{1}{2}\partial_{\hat{m}}\mathcal{B}_{\hat{n}\hat{p}}dy^{\hat{m}}\wedge dy^{\hat{n}}\wedge dy^{\hat{p}}.
\end{aligned} \tag{3.7}$$

Finally,

$$\mathcal{V}_{\hat{m}} = \frac{\partial_{\hat{m}}A}{\sqrt{(\partial_{\hat{n}}A)(\partial^{\hat{n}}A)}}, \quad (\mathcal{V}_{\hat{m}}\mathcal{V}^{\hat{m}} = 1), \tag{3.8}$$

where $A(y^{\hat{m}})$ is the auxiliary scalar field introduced by PST to impose the self-duality of the tensor $H_{\hat{m}\hat{n}\hat{p}}$ as an equation of motion.

Since we will consider only four-dimensional contributions with up to two derivatives, it will be sufficient to write

$$\sqrt{-\det(\hat{g}_{\hat{m}\hat{n}} + iH^*_{\hat{m}\hat{n}})} \simeq \sqrt{-\hat{g}} \left(1 - \frac{1}{4}H^*_{\hat{m}\hat{n}}H^*_{\hat{m}\hat{n}}\right). \tag{3.9}$$

The action (3.5) simplifies then to

$$\begin{aligned}
S_{M5} &= -T_5 \int_{\mathcal{W}_6} d^6y \sqrt{-\hat{g}} \left(\frac{1}{4}\mathcal{V}_{\hat{l}}H^{*\hat{l}\hat{m}\hat{n}}(H_{\hat{m}\hat{n}\hat{p}} - H^*_{\hat{m}\hat{n}\hat{p}})\mathcal{V}^{\hat{p}} + 1 \right) \\
&\quad - T_5 \int_{\mathcal{W}_6} \left(\hat{C}_6 - \frac{1}{2}d\mathcal{B}_2 \wedge \hat{C}_3 \right).
\end{aligned} \tag{3.10}$$

The PST formalism possesses various local symmetries. One of them allows a gauge choice in which $A(y^{\hat{m}})$ is a function of y and \bar{y} only, so that

$$\mathcal{V}^{\mu} = 0, \quad \mathcal{V}_4\mathcal{V}^4 + \mathcal{V}_5\mathcal{V}^5 = 1, \tag{3.11}$$

which preserves four-dimensional Lorentz covariance. With our truncation (3.3) of the five-brane excitations and of the bulk fields, we are led to only retain components

$$\begin{aligned}
H_{\mu\hat{4}\hat{5}} &= \partial_{\mu}\mathcal{B}_{\hat{4}\hat{5}} - \hat{C}_{\mu\hat{4}\hat{5}}, & H_{\mu\nu\rho} &= 3\partial_{[\mu}\mathcal{B}_{\nu\rho]} - \hat{C}_{\mu\nu\rho}, \\
\mathcal{B}_{\hat{4}\hat{5}} &\equiv k^2\mathcal{B}, & \hat{C}_{\mu\hat{4}\hat{5}} &= k^2a(x)\partial_{\mu}X, & \hat{C}_{\mu\nu\rho} &= 3C_{[\mu\nu 4}\partial_{\rho]}X,
\end{aligned} \tag{3.12}$$

where $a(x)$ is defined by $C_{i\bar{j}4} = ia(x)\delta_{i\bar{j}}$. In addition, our reduction of the eleven-dimensional space-time metric (A.1) implies that

$$\sqrt{-\hat{g}} \simeq k^2 e^{-2\gamma-3\sigma} \left(1 + \frac{1}{2}e^{3\gamma}(\partial_{\mu}X)(\partial^{\mu}X)\right), \tag{3.13}$$

where $e^2 = -\det(g_{\mu\nu})$ is now the determinant of the four-dimensional space-time metric.

The reduction of the term involving the six-form field follows from two facts. Firstly, with the embedding (3.2) of \mathcal{W}_6 into \mathcal{M}_{11} , one can write

$$\hat{C}_{\mu\nu\rho\sigma\hat{4}\hat{5}} = -i \frac{\partial z^i}{\partial y} \frac{\partial \bar{z}^{\bar{j}}}{\partial \bar{y}} \left[\langle C \rangle_{\mu\nu\rho\sigma i\bar{j}} + C_{\mu\nu\rho\sigma i\bar{j}} + 4(\partial_{[\mu} X) C_{4\nu\rho\sigma] i\bar{j}} \right],$$

where $\langle C \rangle_{\mu\nu\rho\sigma i\bar{j}}$ is the background contribution discussed in paragraph 2.3 and $C_{\mu\nu\rho\sigma i\bar{j}}$ is the field fluctuation. Notice that the equations defining this background involve the reduced eleven-dimensional metric¹³ and $\langle C \rangle_{\mu\nu\rho\sigma i\bar{j}}$ does depend on the metric moduli σ and γ . Secondly, since C_6 is even under \mathbf{Z}_2 , $C_{4\nu\rho\sigma i\bar{j}}$ is cancelled by the O_7 reduction and the component $C_{\mu\nu\rho\sigma i\bar{j}}$ generates the background equation and can be omitted in the four-dimensional effective Lagrangian.

The four-dimensional five-brane action reads then

$$\begin{aligned} S_{M5} &= \int_{\mathcal{M}_4} d^4x \mathcal{L}_{M5}, \\ \mathcal{L}_{M5} &= -\frac{\tilde{T}}{2} \left[\frac{1}{3!} e e^{\gamma+3\sigma} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{3!} \epsilon^{\mu\nu\rho\sigma} \left(\partial_\mu \mathcal{B} - (\partial_\mu X) a \right) H_{\nu\rho\sigma} \right. \\ &\quad - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} (\partial_\mu \mathcal{B}) (\partial_\nu X) C_{\rho\sigma 4} - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} a (\partial_\mu \mathcal{B}_{\nu\rho}) (\partial_\sigma X) \\ &\quad \left. + e e^{\gamma-3\sigma} (\partial_\mu X) (\partial^\mu X) + 2e (e^{-2\gamma-3\sigma} + \langle C \rangle) \right]. \end{aligned} \quad (3.14)$$

This derivation uses

$$T_5 \int_{\mathcal{W}_6} d^6y \sqrt{-\hat{g}} (\dots) = \tilde{T} \int_{\mathcal{M}_4} d^4x e e^{-2\gamma-3\sigma} \left(1 + \frac{1}{2} e^{3\gamma} (\partial_\mu X) (\partial^\mu X) \right) (\dots),$$

where

$$\frac{\tilde{T}}{T_5} = \int_{\mathcal{C}_2} dy d\bar{y} \frac{\partial z^i}{\partial y} \frac{\partial \bar{z}^{\bar{j}}}{\partial \bar{y}} \delta_{i\bar{j}}$$

is the volume of the holomorphic two-cycle in the Calabi-Yau manifold, and the definition $\langle C \rangle_{\mu\nu\rho\sigma i\bar{j}} = i e \epsilon_{\mu\nu\rho\sigma} \langle C \rangle \delta_{i\bar{j}}$. The scalar field \mathcal{B} acts as a Lagrange multiplier. It imposes the constraint

$$\epsilon^{\mu\nu\rho\sigma} \partial_\mu (H_{\nu\rho\sigma} + 3(\partial_\nu X) C_{\rho\sigma 4}) = \epsilon^{\mu\nu\rho\sigma} \partial_\mu (H_{\nu\rho\sigma} + \hat{C}_{\nu\rho\sigma}) = 0.$$

¹³In particular in the Hodge dual.

Its solution is the second eq. (3.12). We can then consider the Lagrangian (3.14) as a function of the unconstrained fields $H_{\mu\nu\rho}$, X and $\hat{\mathcal{B}} = \mathcal{B} - Xa$:

$$\begin{aligned}\mathcal{L}_{\text{M5}} = & -\frac{\tilde{T}}{2} \left[\frac{1}{3!} e^{\gamma+3\sigma} H_{\mu\nu\rho} H^{\mu\nu\rho} + e^{\gamma-3\sigma} (\partial_\mu X)(\partial^\mu X) \right. \\ & + \frac{1}{3} X \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} (\partial_\sigma a) + \frac{1}{2} X^2 \epsilon^{\mu\nu\rho\sigma} (\partial_\mu a)(\partial_\nu C_{\rho\sigma 4}) \\ & \left. - \frac{1}{3!} \epsilon^{\mu\nu\rho\sigma} (\partial_\mu \hat{\mathcal{B}}) (H_{\nu\rho\sigma} - 3X(\partial_\nu C_{\rho\sigma 4})) + 2e(e^{-2\gamma-3\sigma} + \langle C' \rangle) \right].\end{aligned}\quad (3.15)$$

The last term seems to indicate the presence of a scalar potential. However, solving the equation defining the six-form background field shows a cancellation: the scalar potential vanishes as expected by the stability of the configuration which is protected by the residual supersymmetry [27]. We will see in paragraph 3.3 that the supermultiplet structure required to supersymmetrize this bosonic action does not allow the presence of a scalar potential.

The Lagrangians (3.14) and (3.15) are invariant under the residual symmetries:

$$\begin{aligned}\delta C_{\mu\nu 4} &= 2\partial_{[\mu} \Lambda_{\nu]}, \\ \delta a &= c, \quad \delta \mathcal{B} = cX, \quad \delta \hat{\mathcal{B}} = 0, \quad c = \text{constant}.\end{aligned}\quad (3.16)$$

Note moreover that \mathcal{B} appears in the Lagrangian (3.14) only through its derivatives, so the independent symmetry

$$\delta \mathcal{B} = c', \quad c' = \text{constant}, \quad (3.17)$$

is also present. Solving for $\hat{\mathcal{B}}$ in eq. (3.15) leads to a Lagrangian for $\mathcal{B}_{\mu\nu}$ and X , which will be supersymmetrized using a linear multiplet. And solving for $H_{\mu\nu\rho}$ leads to a theory containing a chiral multiplet with scalar components X and $\hat{\mathcal{B}}$. This chiral-linear duality is the four-dimensional consequence of the self-duality of the brane three-index tensor $H_{\hat{m}\hat{n}\hat{p}}$, when expressed in the covariant formalism of PST.

We now consider the supersymmetrization in four space-time dimensions of the reduced five-brane Lagrangian, first without supergravity background, then with the coupling to the eleven-dimensional background fields.

3.2 Supersymmetrization without supergravity background

Our first goal is to identify the supermultiplet content of the effective four-dimensional supergravity expected to arise from our truncation of the five-brane spectrum. The

simplest procedure is to consider the flat, zero-background limit of the five-brane Lagrangian (3.15), which becomes

$$\mathcal{L}_{\text{M5, flat}} = -\frac{\tilde{T}}{2} \left[\frac{1}{3!} H_{\mu\nu\rho} (H^{\mu\nu\rho} + \epsilon^{\mu\nu\rho\sigma} \partial_\sigma \mathcal{B}) + (\partial_\mu X)(\partial^\mu X) \right]. \quad (3.18)$$

Introducing for convenience the four-dimensional vector field

$$v^\mu = \frac{1}{3!} \epsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma}, \quad (3.19)$$

we obtain

$$\mathcal{L}_{\text{M5, flat}} = \frac{\tilde{T}}{2} \left[v^\mu (\partial_\mu \mathcal{B} + v_\mu) - (\partial_\mu X)(\partial^\mu X) \right]. \quad (3.20)$$

Solving for v_μ leads to $v_\mu = -\frac{1}{2} \partial_\mu \mathcal{B}$, so that

$$\mathcal{L}_{\text{M5, flat}} = -\frac{\tilde{T}}{2} \left[\frac{1}{4} (\partial_\mu \mathcal{B})(\partial^\mu \mathcal{B}) + (\partial_\mu X)(\partial^\mu X) \right]. \quad (3.21)$$

Alternatively, solving for \mathcal{B} gives

$$\partial_\mu v^\mu = 0 \quad \longrightarrow \quad H_{\mu\nu\rho} = 3\partial_{[\mu} \mathcal{B}_{\nu\rho]}, \quad (3.22)$$

and we obtain the equivalent form of the Lagrangian

$$\mathcal{L}_{\text{M5, flat}} = -\frac{\tilde{T}}{2} \left[\frac{1}{3!} H^{\mu\nu\rho} H_{\mu\nu\rho} + (\partial_\mu X)(\partial^\mu X) \right]. \quad (3.23)$$

This discussion illustrates again how the six-dimensional self-duality condition on $H_{\hat{m}\hat{n}\hat{p}}$ translates in the truncated four-dimensional theory into a duality equivalence of an antisymmetric tensor $\mathcal{B}_{\mu\nu}$ with a (pseudo)scalar \mathcal{B} .

We now observe that expression (3.20) is precisely the bosonic part of the supersymmetric Lagrangian

$$\mathcal{L}_{\text{flat}} = -\tilde{T} \int d^2\theta d^2\bar{\theta} \left(\hat{V}^2 - \frac{1}{2} (\hat{S} + \bar{\hat{S}}) \hat{V} \right), \quad (3.24)$$

where \hat{V} is a real vector superfield and \hat{S} is a chiral superfield. Using the component expansions

$$\begin{aligned} \hat{V} &= \hat{C} + (\theta\sigma^\mu\bar{\theta})\hat{v}_\mu + \theta\theta(\hat{m} + i\hat{n}) + \bar{\theta}\bar{\theta}(\hat{m} - i\hat{n}) \\ &\quad + \theta\theta\bar{\theta}\bar{\theta}(\hat{d} - \frac{1}{4}\square\hat{C}) + \dots, \\ \hat{S} &= \hat{s} - \theta\theta\hat{f}_s - i(\theta\sigma^\mu\bar{\theta})\partial_\mu\hat{s} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square\hat{s} + \dots, \end{aligned} \quad (3.25)$$

where the dots indicate fermion contributions, the bosonic part of the supersymmetric Lagrangian (3.24) is

$$\begin{aligned}\mathcal{L}_{\text{flat, bos.}} = & \frac{\tilde{T}}{2} \left[\hat{v}^\mu (\hat{v}_\mu - \partial_\mu \text{Im } \hat{s}) - (\partial_\mu \hat{C})(\partial^\mu \hat{C}) \right. \\ & \left. - 2\hat{d}(2\hat{C} - \text{Re } \hat{s}) - 4(\hat{m}^2 + \hat{n}^2) - (\hat{f}_s(\hat{m} - i\hat{n}) + \text{c.c.}) \right],\end{aligned}\quad (3.26)$$

omitting a space-time derivative. The second line is auxiliary and vanishes when solving for either $\text{Re } \hat{s}$ and \hat{f}_s or \hat{m} , \hat{n} , \hat{d} and \hat{f}_s . The first line is eq. (3.20).

The chiral-linear duality present in the globally supersymmetric Lagrangian (3.24) is the consequence, in the truncated theory, of the self-duality property of the five-brane antisymmetric tensor. Explicitly, solving for the vector superfield \hat{V} in eq. (3.24) leads to $\hat{V} = \frac{1}{4}(\hat{S} + \bar{\hat{S}})$. For the bosonic components, this is $\hat{C} = \frac{1}{2} \text{Re } \hat{s}$, $\hat{v}_\mu = \frac{1}{2}(\partial_\mu \text{Im } \hat{s})$ and $\hat{m} + i\hat{n} = -\frac{1}{4}\hat{f}_s$. The supersymmetric Lagrangian becomes then

$$\mathcal{L}_{\text{flat}} = \frac{\tilde{T}}{8} \int d^2\theta d^2\bar{\theta} \hat{S} \bar{\hat{S}} = -\frac{\tilde{T}}{8} \left[(\partial_\mu \hat{S})(\partial^\mu \bar{\hat{S}}) - \hat{f}_s \bar{\hat{f}}_s \right] + \text{fermionic terms}.$$

Alternatively, we can rewrite expression (3.24) as

$$\mathcal{L}_{\text{flat}} = -\tilde{T} \int d^2\theta d^2\bar{\theta} \hat{V}^2 - \frac{\tilde{T}}{8} \int d^2\theta \hat{S} \bar{\mathcal{D}} \bar{\mathcal{D}} \hat{V} - \frac{\tilde{T}}{8} \int d^2\bar{\theta} \bar{\hat{S}} \mathcal{D} \mathcal{D} \hat{V},$$

and solve for the chiral superfield \hat{S} , implying that \hat{V} is a real linear multiplet \hat{L} , $\bar{\mathcal{D}} \bar{\mathcal{D}} \hat{L} = \mathcal{D} \mathcal{D} \hat{L} = 0$. For the bosonic components, solving for \hat{s} and \hat{f}_s in expression (3.26) leads to $\hat{d} = \hat{m} = \hat{n} = 0$ and

$$\partial^\mu \hat{v}_\mu = 0 \quad \longrightarrow \quad \hat{v}_\mu = -\frac{1}{3!} \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial^\nu b^{\rho\sigma}.$$

The Lagrangian becomes

$$\mathcal{L}_{\text{flat}} = -\tilde{T} \int d^2\theta d^2\bar{\theta} \hat{L}^2 = -\frac{\tilde{T}}{2} \left[(\partial_\mu \hat{C})(\partial^\mu \hat{C}) + \frac{1}{3!} H^{\mu\nu\rho} H_{\mu\nu\rho} \right] + \text{fermionic terms}.$$

3.3 Supersymmetrization with supergravity background

We now turn on the supergravity background and return to Lagrangian (3.15) to derive its supersymmetric extension.

The description in terms of superconformal multiplets of the supergravity bulk fields has been discussed in detail in ref. [9]. The dilaton and universal modulus are respectively described by two vector multiplets, V with weights $\omega = 2$, $n = 0$ and V_T

with zero weights. Bianchi identities in \mathcal{M}_{11} would constrain V to be linear and V_T to be $T + \bar{T}$ in terms of a chiral multiplet T . Writing the (bosonic) component expansions as

$$\begin{aligned} V &= (C, 0, H, K, v_\mu, 0, d - \square C - \tfrac{1}{3}CR), \\ V_T &= (C_T, 0, H_T, K_T, T_\mu, 0, d_T - \square C_T), \end{aligned}$$

the identification is

$$\begin{aligned} 4\kappa^2 C &= \frac{\lambda^2}{V_6} e^{-3\sigma}, & 4\kappa^2 v_\mu &= \frac{\lambda^2}{V_6} \frac{e}{2} \epsilon_{\mu\nu\rho\sigma} \partial^\nu C^{\rho\sigma 4}, \\ C_T &= 2\frac{\lambda^2}{V_6} e^\gamma, & T_\mu &= -2\frac{\lambda^2}{V_6} \partial_\mu a. \end{aligned} \quad (3.27)$$

Since we may redefine the dimensionless quantity λ^2/V_6 by a scaling of the moduli, we take $\lambda^2/V_6 = 1$ from here on. To describe the five-brane degrees of freedom, we introduce as in the previous paragraph two supermultiplets: a vector supermultiplet \hat{V} and a chiral supermultiplet \hat{S} . We choose them with zero conformal and chiral weights ($\omega = n = 0$). Their bosonic component expansions are

$$\begin{aligned} \hat{V} &= (\hat{C}, 0, \hat{H}, \hat{K}, \hat{v}_\mu, 0, \hat{d} - \square \hat{C}), \\ \hat{S} &= (\hat{s}, 0, -\hat{f}_s, i\hat{f}_s, i\partial_\mu \hat{s}, 0, 0). \end{aligned}$$

To bring the Lagrangian (3.15) in a form appropriate for supersymmetrization in terms of \hat{V} , \hat{S} , V and V_T , we observe that the dimensions of the brane fields $\hat{\mathcal{B}}$, X and $H_{\mu\nu\rho}$ (which are -1 , -1 and 0) do not fit with those of components \hat{s} , \hat{C} and \hat{v}_μ (0 , 0 and 1). Since the only scale in our four-dimensional Poincaré supergravity should be κ , we first introduce a dimensionless five-brane coupling constant

$$\tilde{T} = \frac{\tau}{\kappa^4}, \quad (3.28)$$

and perform the rescalings

$$H_{\mu\nu\rho} = \kappa \tilde{H}_{\mu\nu\rho}, \quad X = \kappa \tilde{X}, \quad \hat{\mathcal{B}} = \kappa \tilde{\mathcal{B}}. \quad (3.29)$$

Action (3.15) rewrites then as

$$\begin{aligned} \mathcal{L}_{M5} &= -\frac{\tau}{2\kappa^2} \left[\frac{1}{3!} e^{\gamma+3\sigma} \tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho} + e^{\gamma-3\sigma} (\partial_\mu \tilde{X}) (\partial^\mu \tilde{X}) \right. \\ &\quad + \frac{1}{3} \tilde{X} \epsilon^{\mu\nu\rho\sigma} \tilde{H}_{\mu\nu\rho} (\partial_\sigma a) + \frac{1}{2} \tilde{X}^2 \epsilon^{\mu\nu\rho\sigma} (\partial_\mu a) (\partial_\nu C_{\rho\sigma 4}) \\ &\quad \left. - \frac{1}{3!} \epsilon^{\mu\nu\rho\sigma} (\partial_\mu \tilde{\mathcal{B}}) (\tilde{H}_{\nu\rho\sigma} - 3\tilde{X} (\partial_\nu C_{\rho\sigma 4})) \right] - V_0, \end{aligned} \quad (3.30)$$

with an apparent scalar potential

$$V_0 = \frac{\tau}{\kappa^4} e (e^{-2\gamma-3\sigma} + \langle C \rangle). \quad (3.31)$$

Then, to go to the superconformal formalism, we recall [9] that $\frac{1}{\kappa^2}$ is the Poincaré gauge-fixed value of the multiplet¹⁴

$$\Upsilon = (S_0 \bar{S}_0 V_T)^{3/2} (2V)^{-1/2}. \quad (3.32)$$

Suppose that we identify the scalar field \tilde{X} with the lowest component \hat{C} of the brane multiplet \hat{V} . Identifications (3.27) also indicate that $e^{-3\sigma}$ is the lowest component of $4V\Upsilon^{-1}$ while e^γ is the lowest component of $\frac{1}{2}V_T$. We then infer that the first line of action (3.30) appears in the component expansion of

$$-\tau[VV_T\hat{V}^2]_D,$$

which is independent from Υ . Comparison of the $\hat{v}^\mu \hat{v}_\mu$ term with the $\tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho}$ term in the actions leads then to the identifications

$$\hat{C} = \tilde{X}, \quad \hat{v}_\mu = -\frac{e}{6} \frac{1}{4\kappa^2 C} \epsilon_{\mu\nu\rho\sigma} \tilde{H}^{\nu\rho\sigma}. \quad (3.33)$$

With these results, the vector component of $V\hat{V}$ is then

$$C\hat{v}_\mu + \hat{C}v_\mu = -\frac{1}{4\kappa^2} \frac{e}{3!} \epsilon_{\mu\nu\rho\sigma} (\tilde{H}^{\nu\rho\sigma} - 3\tilde{X}\partial^\nu C^{\rho\sigma 4}),$$

which is the combination appearing in the last line of Lagrangian (3.30). We then conclude that

$$\mathcal{L}_{\text{brane}} = -\tau \left[VV_T \hat{V}^2 - \frac{1}{2} (\hat{S} + \bar{\hat{S}}) V\hat{V} \right]_D \quad (3.34)$$

is the superconformal tensor calculus expression for the five-brane kinetic Lagrangian, with in addition

$$\hat{\mathcal{B}} = \text{Im } \hat{s}. \quad (3.35)$$

Expression (3.34) is independent from the compensating multiplet S_0 and completely frame-independent. Its component expansion does not include any eR term and the

¹⁴The chiral S_0 , with weights $w = n = 1$, is the compensating multiplet: some of its components are used to gauge fix superconformal symmetries to realize Poincaré invariance only.

Einstein frame condition for dilatation symmetry would not be affected by its addition to bulk (and S^1/\mathbf{Z}_2 plane) contributions. The bosonic component expression reads

$$\begin{aligned}
e^{-1}\mathcal{L}_{\text{brane}} = & -\tau CC_T \left((\partial_\mu \hat{C})(\partial^\mu \hat{C}) - \hat{v}_\mu \hat{v}^\mu \right) + 2\tau C \hat{C} \hat{v}^\mu T_\mu + \tau \hat{C}^2 v^\mu T_\mu \\
& + \tau (\partial_\mu \text{Im } \hat{s})(C \hat{v}^\mu + \hat{C} v^\mu) \\
& + \tau \hat{C}^2 (C_T d - C d_T) - 2\tau C \hat{C} (\partial_\mu C_T)(\partial^\mu \hat{C}) - \tau \hat{C}^2 (\partial_\mu C)(\partial^\mu C_T) \quad (3.36) \\
& + \tau (\text{Re } \hat{s} - 2\hat{C} C_T) \left(C \hat{d} + \hat{C} d - v^\mu \hat{v}_\mu + (\partial^\mu C)(\partial_\mu \hat{C}) \right) \\
& + e^{-1}\mathcal{L}_{\text{aux.}} + \text{total derivative.}
\end{aligned}$$

The auxiliary Lagrangian vanishes ‘on-shell’: it is a quadratic expression in H , K , H_T , K_T , \hat{H} , \hat{K} and \hat{f}_s . To compare the above expression with eq. (3.30), we also need to solve for $\text{Re } \hat{s}$, which is not generated by the reduction of the brane bosonic world-volume action: its presence is required by supersymmetry only. The fourth line is then eliminated. All contributions from the third line are related by supersymmetry to propagation of the background fields and are invisible in eq. (3.30). And the first two lines with identifications (3.33) and (3.35) correspond to eq. (3.30), with the exception of the scalar potential V_0 which cannot arise from the superconformal expression (3.34).

As expected from the self-duality of the brane tensor $H_{\hat{m}\hat{n}\hat{p}}$, the supergravity Lagrangian (3.34) has chiral-linear duality. Solving for the vector superfield \hat{V} gives $\hat{V} = \frac{1}{4}V_T^{-1}(\hat{S} + \bar{\hat{S}})$ and we obtain

$$\mathcal{L}_{\text{brane, chiral}} = \frac{\tau}{16} \left[V V_T^{-1} (\hat{S} + \bar{\hat{S}})^2 \right]_D.$$

Alternatively, solving for the scalar superfield \hat{S} leads to $V\hat{V} = \hat{L}$, where \hat{L} is a real linear superfield, and then

$$\mathcal{L}_{\text{brane, linear}} = -\tau \left[V_T V^{-1} \hat{L}^2 \right]_D.$$

Chiral-linear duality requires invariance under $\delta \hat{S} = \text{an imaginary constant}$. This symmetry also excludes a superpotential and then the generation of a scalar potential.

The conclusion is that the superconformal Lagrangian (3.34) provides the four-dimensional effective kinetic Lagrangian for the brane modulus multiplet. As in action (2.20), the complete effective four-dimensional supergravity is the known effective theory of orbifold gauge and matter multiplets plus expression (3.34). Most importantly, the brane contributions to the background equations must be taken into account to correctly evaluate the scalar potential. This is the last point we need to discuss before analysing the complete supergravity theory.

3.4 Background and scalar potential

Returning to the bosonic action (2.20), we observe that the background value of the eleven-dimensional Lagrangian is

$$\langle \mathcal{L} \rangle = \left\langle -\frac{1}{2\kappa_{11}^2} \left[eR + \frac{1}{2} G_4 \wedge *G_4 \right] + \mathcal{L}_{\text{kin.}}(H_3) \wedge \delta_5(\mathcal{W}_6) \right\rangle,$$

for a single brane and omitting gauge field contributions on orbifold planes. In our reduced metric, it is in principle a function of the background scalar fields $\sigma(x^4)$ and $\gamma(x^4)$, and of their first and second derivatives which appear in the curvature scalar R . However, using the conditions imposed by the background Einstein equations, one finds that $\langle \mathcal{L} \rangle$ is a derivative,

$$\langle \mathcal{L} \rangle = \frac{1}{2\kappa_{11}^2} \frac{d}{dx^4} \left[e^{-3\gamma} \frac{d}{dx^4} (\gamma + 2\sigma) \right],$$

which disappears after integration on x^4 : the four-dimensional effective Lagrangian has zero background value. As a result, the effective four-dimensional scalar potential generated by the brane modulus vanishes.

Taking several branes and the orbifold planes into account leads to the same result: the scalar potential vanishes as long as a superpotential is not generated by charged matter chiral superfields.

4 The coupled theory

In compactified M-theory, the presence of the five-brane modulus multiplet does not modify the Bianchi identities verified by the massless components $G_{4\mu\nu\rho}$, $G_{4\mu i \bar{j}}$ and G_{4ijk} . Its effect on the four-dimensional effective supergravity is simply the addition of the kinetic Lagrangian (3.34) and the modification of the background equation (2.22) by the source terms proportional to $\delta_5(\mathcal{W}_{6,j})$. More changes will occur with gauge thresholds and anomaly-cancelling terms, which can be regarded as ‘higher-order’ corrections.

The complete effective supergravity¹⁵ of M-theory compactified on O_7 with a five-

¹⁵Up to terms with two derivatives.

brane can be written as follows:

$$\begin{aligned}
\mathcal{L} = & \left[-(S_0 \overline{S_0} V_T)^{3/2} (2V)^{-1/2} - (S + \overline{S})(V + 2\Omega) + (U(W - \alpha M^3) + \text{c.c.}) \right. \\
& + (L_T - 2 \sum_a \beta^a \Omega^a)(V_T + 2\overline{M} e^A M) + V(\epsilon |\alpha M^3|^2 - 2\delta \overline{M} e^A M) \Big]_D \\
& + [S_0^3 W]_F \\
& - \tau \left[V V_T \hat{V}^2 - \frac{1}{2}(\hat{S} + \overline{\hat{S}}) V \hat{V} \right]_D + \frac{\tau}{4} [\hat{S} \sum_a \hat{\beta}^a \mathcal{W}^a \mathcal{W}^a]_F,
\end{aligned} \tag{4.1}$$

The first three lines [9] collect all contributions from gauge multiplets (in the Chern-Simons superfields Ω^a , $\Omega = \sum_a c^a \Omega^a$, for a gauge group $\prod_a G^a$) and charged matter multiplets¹⁶ M . They also include the contributions of the massless modes of G_4 and of the metric tensor, in the multiplets V , V_T and W . The first term is the bulk Lagrangian [28, 29] produced by the reduction of the CJS theory. The next terms induce by the field equations of the Lagrange multipliers S , U and L_T the Bianchi identities. The solutions are:

$$\begin{aligned}
S \text{ (chiral, } w = n = 0) : \quad & V = L - 2\Omega \quad (L \text{ linear, } w = 2, n = 0), \\
U \text{ (vector, } w = 2, n = 0) : \quad & W = \alpha M^3 + ic \quad (c \text{ real}), \\
L_T \text{ (linear, } w = 2, n = 0) : \quad & V_T = T + \overline{T} - 2\overline{M} e^A M \quad (T \text{ chiral, } w = n = 0).
\end{aligned}$$

Reduction of the action (2.20) shows that the massless components of the six-form field are included in these Lagrange multipliers. The single term in the third line is the superpotential, as defined by the Bianchi identity induced by U . The contributions with coefficients β^a , ϵ and δ are higher-order corrections following from anomaly cancellation. They generate in particular gauge thresholds. This formulation is derived and explained in ref. [9].

The last line in eq. (4.1) is the brane Lagrangian (3.34), supplemented by a higher-order correction with coefficients $\tau \hat{\beta}^a$. Its role will be discussed below. The identity

$$\frac{1}{4} [\hat{S} \mathcal{W}^a \mathcal{W}^a]_F = -2 \left[(\hat{S} + \overline{\hat{S}}) \Omega^a \right]_D + \text{derivative} \tag{4.2}$$

can also be used as a definition of the gauge curvature chiral multiplets \mathcal{W}^a .

Theory (4.1) has a very simple Einstein term since only the bulk Lagrangian contributes:

$$\mathcal{L}_{\text{Einstein}} = -\frac{1}{2} e R \left[(z_0 \overline{z_0} C_T)^{3/2} (2C)^{-1/2} \right], \tag{4.3}$$

¹⁶The chiral multiplet M denotes a generic charged matter multiplet, for instance a **27** of an E_6 gauge group.

where z_0 , C_T and C denote the lowest components of S_0 , V_T and V . As mentioned already in eq. (3.32), the Einstein frame is selected by the condition

$$\left(\frac{z_0 \bar{z}_0 C_T}{2C}\right)^{-3/2} = 2\kappa^2 C. \quad (4.4)$$

The Einstein frame will be used below.

Since theory (4.1) contains ‘auxiliary multiplets’ which can be eliminated, we will consider two versions related by chiral-linear duality acting on the dilaton multiplet:

- The *linear version* is obtained by solving for L_T , U and S . The dynamical multiplets are then L , T , M , Ω^a and the brane multiplet \hat{L} or \hat{S} . The dilaton is described by the linear multiplet L , which also includes the massless component $G_{4\mu\nu\rho}$ of the four-form field.
- The *chiral version* is obtained by solving for L_T , U and V , the dynamical multiplets being then S , T , M , Ω^a and the brane multiplet \hat{L} or \hat{S} . The dilaton is described by the real part $\text{Re } S$ of the scalar component of the chiral multiplet S , while $\text{Im } S$ is a component of the six-form field.

For our purposes, it is useful to simplify the theory by solving for L_T and U . Their field equations respectively imply that $V_T = T + \bar{T} - 2\bar{M}e^A M$, with a chiral modulus multiplet T , and that the superpotential is a cubic gauge invariant function of M which we symbolically write $W(M) = \alpha M^3$, up to a possible constant (which would break supersymmetry). The result is the following effective Lagrangian:

$$\begin{aligned} \mathcal{L} = & \left[-\left(S_0 \bar{S}_0 (T + \bar{T} - 2\bar{M}e^A M) \right)^{3/2} (2V)^{-1/2} - (S + \bar{S})(V + 2\Omega) \right. \\ & \left. + V(\epsilon |\alpha M^3|^2 - 2\delta \bar{M}e^A M) \right]_D - \tau \left[V(T + \bar{T} - 2\bar{M}e^A M) \hat{V}^2 - \frac{1}{2}(\hat{S} + \bar{\hat{S}})V\hat{V} \right]_D \\ & + \left[S_0^3 W(M) + \frac{1}{4} \sum_a (\beta^a T + \tau \hat{\beta}^a \hat{S}) \mathcal{W}^a \mathcal{W}^a \right]_F. \end{aligned} \quad (4.5)$$

We first omit the higher-order corrections: $\beta^a = \hat{\beta}^a = \delta = \epsilon = 0$. All terms in the Lagrangian are then obtained from the reduction of the higher-dimensional bosonic action (2.20) and of the brane action (3.5), supplemented by $N = 1$ supersymmetry. We also choose to describe the brane multiplet by the chiral multiplet \hat{S} by solving for

\hat{V} . Then, with identity (4.2),

$$\begin{aligned} \mathcal{L} = & \left[- \left(S_0 \overline{S}_0 (T + \overline{T} - 2\overline{M} e^A M) \right)^{3/2} (2V)^{-1/2} \right. \\ & \left. - \left(S + \overline{S} - \frac{\tau}{16} \frac{(\hat{S} + \overline{\hat{S}})^2}{T + \overline{T} - 2\overline{M} e^A M} \right) V \right]_D + \left[S_0^3 W(M) + \frac{1}{4} S \sum_a c^a \mathcal{W}^a \mathcal{W}^a \right]_F, \end{aligned} \quad (4.6)$$

and solving for V leads to the chiral version, in which the (bulk) dilaton is described by S . It is as usual defined by

$$\mathcal{L}_{\text{chiral}} = -\frac{3}{2} \left[S_0 \overline{S}_0 e^{-K/3} \right]_D + \left[\frac{1}{4} \sum_a f^a \mathcal{W}^a \mathcal{W}^a + S_0^3 W(M) \right]_F. \quad (4.7)$$

The real Kähler potential is

$$K = -\log \left(S + \overline{S} - \frac{\tau}{16} \frac{(\hat{S} + \overline{\hat{S}})^2}{T + \overline{T} - 2\overline{M} e^A M} \right) - 3 \log (T + \overline{T} - 2\overline{M} e^A M), \quad (4.8)$$

and the holomorphic gauge kinetic functions are simply

$$f^a = c^a S. \quad (4.9)$$

It is important to realize that the brane kinetic terms affect the dilaton Kähler potential, and that this modification cannot be moved into the gauge kinetic function by a holomorphic redefinition of the chiral S : the brane kinetic terms are not harmonic.

Suppose nevertheless that we insist on defining the dilaton as the real quantity

$$\varphi = \text{Re } S - \frac{\tau}{32} \frac{(\hat{S} + \overline{\hat{S}})^2}{T + \overline{T} - 2\overline{M} M}, \quad (4.10)$$

as suggested by the Kähler potential (4.8). The coupling for the gauge group factor G^a can then be written as¹⁷

$$\frac{1}{g_a^2} = \text{Re } f^a = c^a \left(\varphi + \frac{\tau}{32} \frac{(\hat{S} + \overline{\hat{S}})^2}{T + \overline{T} - 2\overline{M} M} \right). \quad (4.11)$$

In this point of view, the brane contribution appears as a correction to the gauge coupling. However, one cannot find a *holomorphic* function f^a with the field variable φ and this choice of dilaton field is not compatible with the supermultiplet structure required when writing the supergravity Lagrangian in the chiral version.

¹⁷It is the ‘wilsonian gauge coupling’.

The addition of the higher-order corrections is straightforward. In the chiral version, the Kähler potential becomes

$$K = -\log\left(S + \bar{S} - \frac{\tau}{16} \frac{(\hat{S} + \bar{\hat{S}})^2}{T + \bar{T} - 2\bar{M}e^A M} + 2\delta\bar{M}e^A M - \epsilon|\alpha M^3|^2\right) - 3\log\left(T + \bar{T} - 2\bar{M}e^A M\right) \quad (4.12)$$

while the gauge kinetic functions read

$$f^a = c^a S + \beta^a T + \tau\hat{\beta}^a \hat{S}. \quad (4.13)$$

The ‘natural’ definition of the dilaton suggested by the Kähler potential is now

$$\varphi = \text{Re } S - \frac{\tau}{32} \frac{(\hat{S} + \bar{\hat{S}})^2}{T + \bar{T} - 2\bar{M}M} + \delta\bar{M}M - \frac{1}{2}\epsilon|\alpha M^3|^2, \quad (4.14)$$

and in terms of this dilaton, the gauge couplings become

$$\begin{aligned} \frac{1}{g_a^2} = & c^a \varphi + \beta^a \text{Re } T - c^a \delta\bar{M}M + \frac{1}{2}c^a \epsilon|\alpha M^3|^2 \\ & + \frac{1}{2}\tau(T + \bar{T} - 2\bar{M}M) \left(\frac{1}{16}c^a \left(\frac{\hat{S} + \bar{\hat{S}}}{T + \bar{T} - 2\bar{M}M} \right)^2 + \hat{\beta}^a \frac{\hat{S} + \bar{\hat{S}}}{T + \bar{T} - 2\bar{M}M} \right). \end{aligned} \quad (4.15)$$

Returning to the Lagrangian (4.1), the field equation relating \hat{V} and \hat{S} is

$$\hat{V} = \frac{1}{4} \frac{\hat{S} + \bar{\hat{S}}}{T + \bar{T} - 2\bar{M}e^A M}, \quad (4.16)$$

and the lowest component \hat{C} of \hat{V} has been identified with $\tilde{X} = \kappa^{-1}X$, which is the brane modulus in the direction x^4 , in Planck units. The gauge couplings can then finally be expressed as

$$\begin{aligned} \frac{1}{g_a^2} = & c^a \varphi + \beta^a \text{Re } T - c^a \delta\bar{M}M + \frac{c^a \epsilon}{2}|\alpha M^3|^2 \\ & + \frac{\tau}{2}(T + \bar{T} - 2\bar{M}M) \left(c^a \tilde{X}^2 + 4\hat{\beta}^a \tilde{X} \right). \end{aligned} \quad (4.17)$$

The linear version is interesting. Solving in eq. (4.5) for S implies $V = L - 2\Omega$, and the resulting effective supergravity reads

$$\begin{aligned} \mathcal{L}_{\text{linear}} = & \left[-\frac{1}{\sqrt{2}} \left(S_0 \bar{S}_0 (T + \bar{T} - 2\bar{M}e^A M) \right)^{3/2} (L - 2\Omega)^{-1/2} \right. \\ & + (L - 2\Omega) \left(\epsilon|\alpha M^3|^2 - 2\delta\bar{M}e^A M - \tau(T + \bar{T} - 2\bar{M}e^A M) \hat{V}^2 \right. \\ & \left. \left. + \frac{\tau}{2}(\hat{S} + \bar{\hat{S}})\hat{V} \right) \right]_D + \left[S_0^3 W(M) + \frac{1}{4} \Sigma_a (\beta^a T + \tau\hat{\beta}^a \hat{S}) \mathcal{W}^a \mathcal{W}^a \right]_F. \end{aligned} \quad (4.18)$$

In this case, with the identification $\hat{C} = \tilde{X}$ and with the field equation (4.16) relating \hat{S} and \hat{C} , computing the gauge couplings leads easily to

$$\begin{aligned} \frac{1}{g_a^2} &= \frac{1}{2}c^a \left(\frac{z_0 \bar{z}_0 (T + \bar{T} - 2\bar{M}M)}{2C} \right)^{3/2} + \frac{1}{2}c^a \epsilon |\alpha M^3|^2 - c^a \delta \bar{M}M \\ &\quad + \frac{\tau}{2}(T + \bar{T} - 2\bar{M}M)[c^a \tilde{X}^2 + 4\hat{\beta}^a \tilde{X}] + \beta^a \text{Re } T. \end{aligned} \quad (4.19)$$

Comparing with expression (4.17), we find that

$$2\varphi = \left(\frac{z_0 \bar{z}_0 (T + \bar{T} - 2\bar{M}M)}{2C} \right)^{3/2}$$

or, in the Einstein frame, with condition (4.4) and $C_T = T + \bar{T} - 2\bar{M}M$,

$$\varphi = \frac{1}{4\kappa^2 C}. \quad (4.20)$$

The compatibility of expressions (4.19) and (4.13) follows then from the field equation of the vector multiplet V in theory (4.5), which is chiral-linear duality:

$$\begin{aligned} S + \bar{S} &= \left(\frac{S_0 \bar{S}_0 (T + \bar{T} - 2\bar{M}e^A M)}{2V} \right)^{3/2} + \epsilon |\alpha M^3|^2 - 2\delta \bar{M}e^A M \\ &\quad - \tau(T + \bar{T} - 2\bar{M}e^A M)\hat{V}^2 + \frac{\tau}{2}(\hat{S} + \bar{\hat{S}})\hat{V}. \end{aligned} \quad (4.21)$$

To summarize, in the chiral version of the effective supergravity, the kinetic Lagrangian of the five-brane modulus introduces a quadratic, non-harmonic correction to the dilaton in the Kähler potential. The holomorphic gauge functions and the wilsonian gauge couplings are not affected by these terms. In the linear version, the kinetic brane Lagrangian generates quadratic, non-harmonic corrections to the field-dependent wilsonian gauge couplings.

The higher-order brane contribution $\Delta_{\text{brane}} = \frac{1}{4}\tau \sum_a \hat{\beta}^a [\hat{S} \mathcal{W}^a \mathcal{W}^a]_F$ is similar to the familiar gauge thresholds in the modulus T , with coefficients β^a . We have seen that the self-duality of the three-index tensor on the brane world-volume leads in four dimensions to a chiral-linear duality. In the effective supergravity, this duality requires invariance under variations of \hat{S} by an imaginary constant. Then, D -terms should depend on $\hat{S} + \bar{\hat{S}}$, and with our set of multiplets, there is a unique F -term compatible with this symmetry: the higher-order correction Δ_{brane} ¹⁸.

¹⁸The authors of refs. [30, 31] failed to recognize the importance of self-duality of the world-volume three-form. They attempted to describe the brane modulus with a chiral multiplet and introduced a quadratic holomorphic F -density forbidden by chiral-linear duality and unrelated to brane kinetic terms. The resulting supergravity theory is incorrect.

This second brane contribution is not generated by reduction of the PST brane action (3.5), as T -dependent threshold corrections do not follow from reduction of the bosonic action (2.20). In that sense, they can be regarded as higher-order terms.

The presence of quadratic and linear brane contributions has been established in the background calculation of Lukas, Ovrut and Waldram [22, 6, 30]. They have in particular computed the gauge couplings for a set of branes located at fixed positions along S^1 . These positions correspond to constant background values of our scalar field \tilde{X} . To compare with our result, it is easier (and sufficient) to consider a single brane, two gauge couplings and a single modulus T , as in our reduction. The variables used by LOW are then the position z along the interval S^1/\mathbf{Z}_2 and three charges $\beta^{(0)}$, $\beta^{(2)}$ and $\beta^{(5b.)}$ associated with the two fixed planes and the brane. The variable z is normalized in the interval $[0, 1]$ and the charges are quantized: $\beta^{(0)}$ and $\beta^{(2)}$ are half integers, $\beta^{(5b.)}$ is an integer, and the cohomology (or background) condition implies $\beta^{(0)} + \beta^{(2)} + \beta^{(5b.)} = 0$. The gauge couplings found by LOW are then

$$\begin{aligned}\frac{1}{g_1^2} &= \text{Re } S + \frac{\epsilon_S}{8\pi} \text{Re } T [\beta^{(0)} + (1-z)^2 \beta^{(5b.)}], \\ \frac{1}{g_2^2} &= \text{Re } S + \frac{\epsilon_S}{8\pi} \text{Re } T [-(\beta^{(0)} + \beta^{(5b.)}) + z^2 \beta^{(5b.)}], \\ \frac{1}{g_1^2} - \frac{1}{g_2^2} &= \frac{\epsilon_S}{4\pi} \text{Re } T [\beta^{(0)} + \beta^{(5b.)} - z \beta^{(5b.)}],\end{aligned}\tag{4.22}$$

with a dimensionless (arbitrary) parameter ϵ_S related to the Calabi-Yau volume and the S^1 radius. Compare these quantities with our expression (4.17), with $M = 0$ and $c^a = 1$:

$$\begin{aligned}\frac{1}{g_{1,2}^2} &= \varphi + \text{Re } T [\beta^{1,2} + \tau \tilde{X}^2 + 4\tau \hat{\beta}^{1,2} \tilde{X}], \\ \frac{1}{g_1^2} - \frac{1}{g_2^2} &= \text{Re } T [\beta^1 - \beta^2 + 4\tau (\hat{\beta}^1 - \hat{\beta}^2) \tilde{X}].\end{aligned}\tag{4.23}$$

Our parameters are not normalized or quantized. If we merely write $\tilde{X} = \tilde{\lambda} z$, both sets of equations coincide with the trivial statement $\tau = \frac{\epsilon_S}{8\pi\tilde{\lambda}^2} \beta^{(5b.)}$ and the non-trivial relations

$$\beta^1 = -\beta^2 = \frac{\epsilon_S}{8\pi} (\beta^{(0)} + \beta^{(5b.)}), \quad \hat{\beta}_1 = -\frac{\tilde{\lambda}}{2}, \quad \hat{\beta}_2 = 0.\tag{4.24}$$

These equations are predictions obtained from the solution of the background condition which specify in parts our four arbitrary threshold parameters. They are specific properties of M-theory compactified on O_7 . Our effective supergravity reproduces then nicely the background found by LOW. Notice in passing that the dilaton field is incorrectly identified in eqs. (4.22) as the real part of the chiral S . We have seen

that the correct identification is $\varphi = (4\kappa^2 C)^{-1}$, in the linear version of the theory. A background calculation is not sufficient to reach this conclusion: a constant value z of the brane modulus can be the background value of any kind of multiplets (vector, linear, chiral).

4.1 The scalar potential

We close this section by a discussion of the impact of the five-brane modulus on the supergravity scalar potential.

We first use the chiral version, defined by the Kähler potential (4.8) and the superpotential $W(M)$. We concentrate on the potential at $M = 0$ ¹⁹. We however assume that the superpotential can be nonzero in this limit: this is the case if the component G_{4ijk} of the four-form field is a non-zero constant breaking supersymmetry. As usual, the potential (in the Einstein frame) reads

$$\kappa^4 V(S, T, \hat{S}) = e^K W \overline{W} \left[\sum_{IJ} (K_I + W^{-1} W_I) (K^J + \overline{W}^{-1} \overline{W}^J) (K_J^I)^{-1} - 3 \right],$$

where $K_I = \frac{\partial K}{\partial z^I}$, $K^I = (K_I)^* \dots$, and $z_I = (S, T, \hat{S})$. In the absence of the five-brane field \hat{S} , the potential takes the simple form $\kappa^4 V(S, T) = e^K W \overline{W}$. An explicit calculation shows that this result *is not* affected by the contributions of the five-brane modulus. The complete scalar potential at $M = 0$ in the chiral version of the theory is then:

$$\kappa^4 V(S, T, \hat{S}) = \frac{W \overline{W}}{(S + \overline{S} - \frac{\tau}{16} \frac{(\hat{S} + \overline{\hat{S}})^2}{T + \overline{T}})(T + \overline{T})^3}. \quad (4.25)$$

This result can be easily understood in the linear version of the theory, or in the original expression (4.5) of the Lagrangian. The five-brane terms do not include any contribution to the scalar potential: we have discussed this point in paragraph 3.4. In the linear version, the scalar potential is then completely independent from the brane modulus \hat{C} . This statement would remain true with several five-branes, since each contributes by adding to eq. (4.5) a similar term, without any scalar potential.

The appearance of a dependence in \hat{S} of the potential in the chiral version follows from the chiral-linear duality equation (4.21). The relation between S and C is modified

¹⁹Since M is a charged field, the potential is always stationary at $M = 0$.

by the five-brane to become

$$S + \overline{S} - \frac{\tau}{16} \frac{(\hat{S} + \overline{\hat{S}})^2}{T + \overline{T}} = \frac{1}{2\kappa^2 C}$$

with $M = 0$ and in the Einstein frame. It is the dependence on C of the scalar potential in the linear version which induces a dependence on \hat{S} in the chiral version. As a consequence, the five-brane modulus does not produce a new minimum equation:

$$\frac{\partial V}{\partial \hat{S}} = -\frac{\tau}{8} \frac{\hat{S} + \overline{\hat{S}}}{T + \overline{T}} \frac{\partial V}{\partial S}$$

and $\frac{\partial V}{\partial S}$ is not zero. The impact of the five-brane modulus on the effective scalar potential is then a simple redefinition of the chiral dilaton field S as a function of the (unchanged) C of the linear multiplet formulation.

5 Concluding remarks

Even if the five-brane is not a perturbative object, it is interesting to consider the brane corrections to the four-dimensional effective supergravity from the perspective of string perturbation theory. The string loop-counting field is our multiplet V with dilaton C , and a n -loop term in the Wilson Lagrangian is characterized by a factor $V^{(3n-1)/2}$ [28]. According to eq. (4.1), the kinetic Lagrangian of the five-brane modulus multiplet is similar to a one-loop correction, linear in V . The origin of this factor is simple: the kinetic terms are normalized by the world-volume induced metric $\sqrt{-\hat{g}} \sim e^{-3\sigma-2\gamma} \sim CC_T^{-2}$. Compare now with the one-loop corrections in the modulus T , which are completely understood in compactifications of heterotic strings. Two kinds of contributions arise [10, 11]. The first is a real gauge-group independent term proportional to the Kähler potential $-3\log(T + \overline{T})$, the ‘Green-Schwarz’ term. The second one is a gauge-group dependent correction which involves a holomorphic function. In the chiral version, the Green-Schwarz term corrects the Kähler potential of the S field, it can be regarded as a wave-function renormalization of this field. The second term is then a correction to the gauge kinetic functions f^a . The similarity with the five-brane contributions in the Lagrangian (4.1) is obvious. In the case of the volume modulus T , the one-loop corrections can be understood in terms of a cancellation of target-space duality anomalies. The analogy suggests that anomalies could also help to understand the structure of our five-brane contributions [32].

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Appendix: Notations and conventions

Coordinates and metrics:

For coordinates, our notation is:

$D = 11$ curved space-time:	x^M	$M = 0, \dots, 10$
$D = 4$ curved space-time:	x^μ	$\mu = 0, 1, 2, 3$
S^1/\mathbf{Z}_2 direction:	x^4	
Calabi-Yau directions, real:	x^a	$a = 5, \dots, 10$
Calabi-Yau complex (Kähler) coordinates:	$z^i, \bar{z}^{\bar{i}}$	$i = 1, 2, 3$
Five-brane world-volume coordinates:	$y^{\hat{m}}$	$\hat{m} = \hat{0}, \dots, \hat{5}$

For reduction purposes, we simply use

$$z^l = \frac{1}{\sqrt{2}} (x^l + ix^{l+3}) , \quad \bar{z}^{\bar{l}} = \frac{1}{\sqrt{2}} (x^l - ix^{l+3}) , \quad l = 1, 2, 3.$$

ϵ_{ijk} is the $SU(3)$ -invariant Calabi-Yau tensor with $\epsilon_{123} = \epsilon_{\bar{1}\bar{2}\bar{3}} = 1$.

The space-time metric has signature $(-, +, +, \dots, +)$. The reduction of the eleven-dimensional metric is defined by

$$g_{MN} = \begin{pmatrix} e^{-\gamma-2\sigma} g_{\mu\nu} & 0 & 0 \\ 0 & e^{2\gamma-2\sigma} & 0 \\ 0 & 0 & e^\sigma \delta_{i\bar{j}} \end{pmatrix}. \quad (\text{A.1})$$

Antisymmetric tensors:

Antisymmetrization of n indices has unit weight:

$$A_{[M_1 \dots M_n]} = \frac{1}{n!} (A_{M_1 \dots M_n} \pm (n! - 1) \text{ permutations}).$$

Differential forms:

For a p -index antisymmetric tensor, we define

$$A_p = \frac{1}{p!} A_{M_1 \dots M_p} dx^{M_1} \wedge \dots \wedge dx^{M_p}.$$

Then,

$$A_p \wedge B_q = \frac{1}{p!q!} A_{M_1 \dots M_p} B_{M_{p+1} \dots M_{p+q}} dx^{M_1} \wedge \dots \wedge dx^{M_{p+q}} = C_{p+q},$$

$$C_{M_1 \dots M_{p+q}} = \frac{(p+q)!}{p!q!} A_{[M_1 \dots M_p} B_{M_{p+1} \dots M_{p+q}]}$$

The exterior derivative is $d = \partial_M dx^M$. The curl $F_{p+1} = dA_p$ of a p -form reads then

$$\begin{aligned} dA_p &= \frac{1}{p!} (\partial_M A_{N_1 \dots N_p}) dx^M \wedge dx^{N_1} \wedge \dots \wedge dx^{N_p} \\ &= \frac{1}{(p+1)!} F_{M_1 \dots M_{p+1}} dx^{M_1} \wedge \dots \wedge dx^{M_{p+1}}, \\ F_{M_1 \dots M_{p+1}} &= (p+1) \partial_{[M_1} A_{M_2 \dots M_{p+1}]} \\ &= \partial_{M_1} A_{M_2 \dots M_{p+1}} \pm p \text{ cyclic permutations.} \end{aligned}$$

The volume form in D space-time dimensions is $dx^{M_1} \wedge \dots \wedge dx^{M_D} = \epsilon^{M_1 \dots M_D} d^D x$.

We use analogous conventions for forms in four space-time dimensions.

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